Solid State Electronics

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(update by 許文瑋)
Figure 9.1 (a) Simplified geometry of a pn junction showing uniformly doped regions. (b) The pn junction showing depletion regions, space charge, and electric field.
Figure 9.2 The pn junction and energy-band diagrams for (a) zero bias and (b) Reverse bias.
\[ V_{bi} = |\phi_{Fp}| + |\phi_{Fn}| \] (9.1)

\[ V_{bi} = V_t \ln \left( \frac{N_a N_d}{n_i^2} \right) \] (9.2)
Figure 9.3 The pn junction and energy-band diagram with a forward bias. The electron and hole diffusion across the space charge region is shown.
Figure 9.6 Conduction-band energy through a pn junction showing the potential barrier and depletion widths.
\[ V_{bi} = V_t \ln \left( \frac{N_a N_d}{n_i^2} \right) \]  \hspace{1cm} (9.2)

\[ \phi_{B0} = (\phi_m - \chi) \]  \hspace{1cm} (9.3)

\[ V_{bi} = \phi_{B0} - \phi_n \]  \hspace{1cm} (9.4)

\[ \frac{n_i^2}{N_a N_d} = \exp \left( -\frac{V_{bi}}{V_t} \right) = \exp \left( -\frac{eV_{bi}}{kT} \right) \]  \hspace{1cm} (9.5)

\[ n_{n0} \approx N_d \]  \hspace{1cm} (9.6)

\[ n_{p0} = \frac{n_i^2}{N_a} \]  \hspace{1cm} (9.7)

\[ n_{p0} = n_{n0} \exp \left( -\frac{eV_{bi}}{kT} \right) \]  \hspace{1cm} (9.8)
Figure 9.7 (a) A pn junction with an applied forward-bias voltage showing the directions of the electric field induced by $V_a$ and the space charge electric field. (b) Energy-band diagram of the forward-biased pn junction.
\[ n_p = n_{n0} \exp\left(\frac{-e(V_{bi} - V_a)}{kT}\right) = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right) \exp\left(\frac{+eV_a}{kT}\right) \] \hspace{1em} (9.9)

\[ n_p = n_{p0} \exp\left(\frac{eV_a}{kT}\right) \] \hspace{1em} (9.10)

\[ p_n = p_{n0} \exp\left(\frac{eV_a}{kT}\right) \] \hspace{1em} (9.11)
Figure 9.8 Excess minority-carrier concentrations at the space charge edges generated by the forward-bias voltage.

\[ n_p(-x_p) = n_{p0} \exp \left( \frac{eV_a}{kT} \right) \]

\[ p_n(x_n) = p_{n0} \exp \left( \frac{eV_a}{kT} \right) \]
Figure 9.9 Excess minority-carrier concentrations at the space charge edges generated by a reverse-bias voltage. For a reverse-bias voltage of \( V_R \geq 0.25 \) V, the minority-carrier concentrations at the space charge edges are essentially zero.
\[
D_p \frac{\partial^2 (\delta p_n)}{\partial x^2} - \mu_p \varepsilon \frac{\partial (\delta p_n)}{\partial x} + g' - \frac{\delta p_n}{\tau_{p0}} = \frac{\partial (\delta p_n)}{\partial t} \quad (9.12)
\]

\[
\frac{d^2 (\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0 \quad (x > x_n) \quad (9.13)
\]

\[
\frac{d^2 (\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = 0 \quad (x > x_p) \quad (9.14)
\]

\[p_n(x_n) = p_{n0} \exp\left(\frac{eV_a}{kT}\right) \quad (9.15a)\]

\[n_p(-x_p) = n_{p0} \exp\left(\frac{eV_a}{kT}\right) \quad (9.15b)\]

\[p_n(x \to +\infty) = p_{n0} \quad (9.15c)\]

\[n_p(x \to -\infty) = n_{p0} \quad (9.15d)\]

\[\delta p_n(x) = p_n(x) - p_{n0} = A e^{x/L_p} + B e^{-x/L_p} \quad (x \geq x_n) \quad (9.16)\]

\[\delta n_p(x) = n_p(x) - n_{p0} = C e^{x/L_n} + D e^{-x/L_n} \quad (x \leq -x_p) \quad (9.17)\]
Figure 9.10 Steady-state minority-carrier concentrations in a pn junction under forward bias.

\[ \delta p_n(x) = p_n(x) - p_{n0} = p_{n0} \left[ \exp\left(\frac{eV_a}{kT} \right) - 1 \right] \exp\left(\frac{x_n - x}{L_p}\right) \]  \hspace{1cm} (9.18)

\[ \delta n_p(x) = n_p(x) - n_{p0} = n_{p0} \left[ \exp\left(\frac{eV_a}{kT} \right) - 1 \right] \exp\left(\frac{x_p + x}{L_n}\right) \]  \hspace{1cm} (9.19)
Figure 9.11 Electron and hole current densities through the space charge region of a pn junction.

\[ J_{\text{Total}} = J_p(x_n) + J_n(-x_p) \]

\[ J_p(x_n) = -eD_p \frac{dp_n(x)}{dx} \Bigg|_{x=x_n} \] (9.20)
\[ J_p (x_n) = -eD_p \frac{d(\delta p_n (x))}{dx} \bigg|_{x=x_n} \]  
(9.21)

\[ J_p (x_n) = \frac{eD_p p_{n0}}{L_p} \left[ \exp \left( \frac{eV_a}{kT} \right) - 1 \right] \]  
(9.22)

\[ J_n (-x_p) = eD_n \frac{d(\delta n_p (x))}{dx} \bigg|_{x=-x_p} \]  
(9.23)

\[ J_n (-x_p) = \frac{eD_n n_{p0}}{L_n} \left[ \exp \left( \frac{eV_a}{kT} \right) - 1 \right] \]  
(9.24)

\[ J = J_p (x_n) + J_n (-x_p) = \left( \frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n} \right) \left[ \exp \left( \frac{eV_a}{kT} \right) - 1 \right] \]  
(9.25)
Figure 9.12 Ideal $I - V$ characteristic of a pn junction diode.

\[ J_s = \left( \frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n} \right) \quad (9.26) \quad J = J_s \left[ \exp\left( \frac{eV_a}{kT} \right) - 1 \right] \quad (9.27) \]
Figure 9.13 Ideal $I - V$ characteristic of a pn junction diode with the current plotted on a log scale.
Figure 9.14 Ideal electron and hole current components through a pn junction under forward bias.

\[ J_p(x) = \frac{eD_p p_n n_0}{L_p} \left[ \exp \left( \frac{eV_a}{kT} \right) - 1 \right] \exp \left( \frac{x_n - x}{L_p} \right) \quad (x \geq x_n) \quad (9.28) \]

\[ J_n(x) = \frac{eD_n n_p n_0}{L_n} \left[ \exp \left( \frac{eV_a}{kT} \right) - 1 \right] \exp \left( \frac{x_p + x}{L_n} \right) \quad (x \leq -x_p) \quad (9.29) \]
Figure 9.15 Schematic of temperature effects in a pn junction diode under (a) reverse bias and (b) forward bias.

\[ J_s \propto n_i^2 \propto (T)^3 \exp \left( \frac{-E_g}{kT} \right) \quad \text{(eg*1)} \]

\[ \frac{J_s(310\text{K})}{J_s(300\text{K})} = 4.46 \quad \text{(eg*2)} \]
The “Short” Diode

Figure 9.16 Geometry of a “short” diode.

\[ p_n(x = x_n + W_n) = p_{n0} \quad (9.30) \]

\[ \delta p_n(x) = p_{n0} \left[ \exp \left( \frac{eV_a}{kT} \right) - 1 \right] \frac{\sinh \left( \frac{(x_n + W_n - x)/L_p}{W_n/L_p} \right)}{\sinh \left( W_n/L_p \right)} \quad (9.31) \]

\[ \sinh \left( \frac{x_n + W_n - x}{L_p} \right) \approx \left( \frac{x_n + W_n - x}{L_p} \right) \quad (9.32a) \]

\[ \sinh \left( \frac{W_n}{L_p} \right) \approx \left( \frac{W_n}{L_p} \right) \quad (9.32b) \]

\[ \delta p_n(x) = p_{n0} \left[ \exp \left( \frac{eV_a}{kT} \right) - 1 \right] \left( \frac{x_n + W_n - x}{W_n} \right) \quad (9.33) \]

\[ J_p(x) = \frac{eD_p p_{n0}}{W_n} \left[ \exp \left( \frac{eV_a}{kT} \right) - 1 \right] \quad (9.34) \]

The “short” dopde
Table 9.2 Summary of hole current density expressions for particular n-region lengths
(Geometry shown in Figure 9.16)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Hole Current Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Long” diode ($W_n \gg L_p$)</td>
<td>$J_p(x) = \frac{eD_p p_{n0}}{L_p} \left[ \exp \left( \frac{eV_a}{kT} \right) - 1 \right] \exp \left( \frac{x_n - x}{L_p} \right)$</td>
</tr>
<tr>
<td>“Medium” diode ($W_n \approx L_p$)</td>
<td>$J_p(x) = \frac{eD_p p_{n0}}{L_p} \left[ \exp \left( \frac{eV_a}{kT} \right) - 1 \right] \times \frac{\cosh \left[ (x_n + W_n - x) / L_p \right]}{\sinh \left( W_n / L_p \right)}$</td>
</tr>
<tr>
<td>“Short” diode ($W_n \ll L_p$)</td>
<td>$J_p(x) = \frac{eD_p p_{n0}}{L_p} \left[ \exp \left( \frac{eV_a}{kT} \right) - 1 \right]$</td>
</tr>
</tbody>
</table>
Figure 9.17 Energy-band diagram of a forward biased metal-semiconductor junction.
\[ J_{s \rightarrow m} = e \int_{E_c}^{\infty} \nu_x \, dn \] \hspace{1cm} (9.35)

\[ dn = g_c (E) f_F (E) dE \] \hspace{1cm} (9.36)

\[ dn = \frac{4 \pi (2 m_n^*)^{3/2}}{h^3} \sqrt{E - E_c} \exp \left[ - \frac{(E - E_F)}{kT} \right] dE \] \hspace{1cm} (9.37)

\[ \frac{1}{2} m_n^* \nu^2 = E - E_c \] \hspace{1cm} (9.38)

\[ J = J_{s \rightarrow m} - J_{m \rightarrow s} \] \hspace{1cm} (9.39)
\[ J = \left[ A^* T^2 \exp\left( -\frac{e\phi_{Bn}}{kT} \right) \right] \left[ \exp\left( \frac{eV_a}{kT} \right) - 1 \right] \] (9.40)

\[ A^* \equiv \frac{4\pi e m^* k^2}{h^3} \] (9.41)

\[ J = J_{sT} \left[ \exp\left( \frac{eV_a}{kT} \right) - 1 \right] \] (9.42)

\[ J_{sT} = A^* T^2 \exp\left( -\frac{e\phi_{Bn}}{kT} \right) \] (9.43)
Figure 9.18 Comparison of forward-bias $I-V$ characteristics between a Schottky diode and a pn junction diode.
\[ I_D = I_s \left[ \exp\left( \frac{eV_a}{kT} \right) - 1 \right] \] (9.44)

\[ g_d = \left. \frac{dI_D}{dV_a} \right|_{V_a=V_0} \] (9.45)

\[ r_d = \left. \frac{dV_a}{dI_D} \right|_{I_D=I_{DQ}} \] (9.46)

\[ g_d = \left. \frac{dI_D}{dV_a} \right|_{V_a=V_0} = \left( \frac{e}{kT} \right) I_s \exp\left( \frac{eV_0}{kT} \right) \approx \frac{I_{DQ}}{V_t} \] (9.47)

\[ r_d = \frac{V_t}{I_{DQ}} \] (9.48)
Figure 9.20 (a) A pn junction with an ac voltage superimposed on a forward-biased ac value; (b) the hole concentration at the space charge edge versus time; and (c) the hole concentration versus distance in the n region at three different times (for relatively low ac voltage frequency).
\[ Y = \left( \frac{1}{V_t} \right) (I_{p0} + I_{n0}) + j\omega \left[ \left( \frac{1}{2V_t} \right) (I_{p0}\tau_{p0} + I_{n0}\tau_{n0}) \right] \]  

(9.49)

\[ Y = g_d + j\omega C_d \]  

(9.50)

\[ g_d = \left( \frac{1}{V_t} \right) (I_{p0} + I_{n0}) = \frac{I_{DQ}}{V_t} \]  

(9.51)

\[ C_d = \left( \frac{1}{2V_t} \right) (I_{p0}\tau_{p0} + I_{n0}\tau_{n0}) \]  

(9.52)
Figure 9.21 (a) Small-signal equivalent circuit of ideal forward-biased pn junction diode; (b) complete small-signal equivalent circuit of pn junction.
Figure 9.22 Forward-bias $I - V$ characteristic of a pn junction diode showing the effect of series resistance.

\[ V_{app} = V_a + I r_s \]  \hspace{1cm} (9.53)
Figure 9.23 Generation process in a reverse-biased pn junction.
\[ R = \frac{C_n C_p N_t (np - n_i^2)}{C_n (n + n') + C_p (p + p')} \]  \hspace{1cm} (9.54)

\[ R = \frac{-C_n C_p N_t n_i^2}{C_n n' + C_p p'} \]  \hspace{1cm} (9.55)

\[ R = \frac{-n_i}{\tau_{p0} + \tau_{n0}} \]  \hspace{1cm} (9.57)

\[ \tau_0 = \frac{\tau_{p0} + \tau_{n0}}{2} \]  \hspace{1cm} (9.58)

\[ R = \frac{-n_i}{2\tau_0} \equiv -G \]  \hspace{1cm} (9.59)

\[ J_{\text{gen}} = \int_0^W eG \, dx \]  \hspace{1cm} (9.60)

\[ J_{\text{gen}} = \frac{en_i W}{2\tau_0} \]  \hspace{1cm} (9.61)

\[ J_R = J_s + J_{\text{gen}} \]  \hspace{1cm} (9.62)
Figure 9.24 Energy-band diagram of a forward-biased pn junction including quasi-Fermi levels.
\[
R = \frac{n p - n_i^2}{\tau_{p0}(n + n') + \tau_{n0}(p + p')}
\]  
(9.63)

\[
n = n_i \exp \left( \frac{E_{Fn} - E_{Fi}}{kT} \right)
\]  
(9.64)

\[
p = n_i \exp \left( \frac{E_{Fi} - E_{Fp}}{kT} \right)
\]  
(9.65)

\[
J_{\text{rec}} = \frac{eWn}{2 \tau_0} \exp \left( \frac{eV}{2 kT} \right) = J_{r0} \exp \left( \frac{eV}{2 kT} \right)
\]  
(9.66)
Figure 9.25 Because of recombination, additional holes from the p region must be injected into the space charge region to establish the minority carrier hole concentration in the n region.

\[ J = J_{\text{rec}} + J_D \]  \hspace{1cm} (9.67)  
\[ J_D = J_s \exp\left(\frac{eV_a}{kT}\right) \]  \hspace{1cm} (9.68)
Figure 9.26 Ideal diffusion, recombination, and total current in a forward-biased pn junction.

Ideal diffusion current, $J_D$ (slope = 1)

Recombination current, $J_{\text{rec}}$ (slope = $\frac{1}{2}$)

Total current

$\ln(J)$

$\ln(J_{R0})$

$\ln(J_s)$

$\frac{eV_a}{kT}$
\[
\ln J_{\text{rec}} = \ln J_{r0} + \frac{eV_a}{2kT} = \ln J_{r0} + \frac{V_a}{2V_t} \quad (9.69a)
\]

\[
\ln J_D = \ln J_s + \frac{eV_a}{kT} = \ln J_s + \frac{V_a}{V_t} \quad (9.69b)
\]

\[
I = I_s \left[ \exp \left( \frac{eV_a}{nkT} \right) - 1 \right] \quad (9.70)
\]
Figure 9.27 (a) Zener breakdown mechanism in a reverse-biased pn junction; (b) avalanche breakdown process in a reverse-biased pn junction.
Figure 9.28 Electron and hole current components through the space charge region during avalanche multiplication.
\[ I_n(W) = M_n I_{n0} \quad (9.71) \]

\[ dI_n(x) = I_n(x)\alpha_n \, dx + I_p(x)\alpha_p \, dx \quad (9.72) \]

\[ \frac{dI_n(x)}{dx} = I_n(x)\alpha_n + I_p(x)\alpha_p \quad (9.73) \]

\[ I = I_n(x) + I_p(x) \quad (9.74) \]

\[ \frac{dI_n(x)}{dx} = (\alpha_p - \alpha_n)I_n(x) = \alpha_p I \quad (9.75) \]
\[ \alpha_n = \alpha_p \equiv \alpha \quad (9.76) \]

\[ I_n(W) - I_n(0) = I \int_0^W \alpha \, dx \quad (9.77) \quad \frac{M_n I_n(0) - I_n(0)}{I} = \int_0^W \alpha \, dx \quad (9.78) \]

\[ I - \frac{1}{M_n} = \int_0^W \alpha \, dx \quad (9.79) \quad \int_0^W \alpha \, dx = 1 \quad (9.80) \]

\[ \varepsilon_{\text{max}} = \frac{eN_d x_n}{\varepsilon_s} \quad (9.81) \quad x_n \approx \left( \frac{2 \varepsilon_s V_R}{e} \frac{1}{N_d} \right)^{1/2} \quad (9.82) \]

\[ V_B = \frac{\varepsilon_s \varepsilon_{\text{crit}}^2}{2eN_B} \quad (9.83) \]
Figure 9.29 Critical electric field at breakdown in a one-sided junction as a function of impurity doping concentrations.

Figure 9.30 Breakdown voltage versus impurity concentration in uniformly doped and linearly graded junctions. 
*(From Sze [13].)*
Figure 9.31 Simple circuit for switching a diode from forward to reverse bias.

\[ I = I_F = \frac{V_F - V_a}{R_F} \quad (9.84) \]
Figure 9.32 (a) Steady-state forward-bias minority-carrier concentrations; (b) minority-carrier concentrations at various times during switching.
Figure 9. 33 Current characteristic versus time during diode switching

\[ I = -I_R \approx \frac{-V_R}{R_R} \]  (9.85)

\[ \text{erf} \sqrt{\frac{t_s}{\tau_{p0}}} = \frac{I_F}{I_F + I_R} \]  (9.86)

\[ t_s \approx \tau_{p0} \ln \left( 1 + \frac{I_F}{I_R} \right) \]  (9.87)

\[ \text{erf} \sqrt{\frac{t_2}{\tau_{p0}}} + \frac{\exp \left( -\frac{t_2}{\tau_{p0}} \right)}{\sqrt{\pi t_2 / \tau_{p0}}} = 1 + 0.1 \left( \frac{I_R}{I_F} \right) \]  (9.88)