Solid State Electronics

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(update by 許文瑋)
\[ J_{drf} = \rho v_d \quad (4.1) \]
\[ J_{p|drf} = (ep)v_{dp} \quad (4.2) \]
\[ F = m_p^* a = e \varepsilon \quad (4.3) \]
\[ v_{dp} = \mu_p \varepsilon \quad (4.4) \]
\[ J_{p|drf} = (ep)v_{dp} = e \mu_p p \varepsilon \quad (4.5) \]
\[ J_{n|drf} = \rho v_{dn} = (-en)v_{dn} \quad (4.6) \]
\[ v_{dn} = -\mu_n \varepsilon \quad (4.7) \]
\[ J_{n|drf} = (-en)(-\mu_n \varepsilon) = e \mu_n n \varepsilon \quad (4.8) \]
\[ J_{drf} = e(\mu_n n + \mu_p p) \varepsilon \quad (4.9) \]
Figure 4.1 (a) Typical random behavior of a hole or electron in a semiconductor with no applied electric field. 
(b) Behavior of a hole in a semiconductor – dotted lines for $\varepsilon = 0$ and soled lines for $\varepsilon > 0$. (c) Behavior of an electron in a semiconductor – dotted lines for $\varepsilon = 0$ and solid lines for $\varepsilon > 0$. 
To get a meaningful and simple result, we take the mean time $= \tau$ and assume after every collision $v=0$, then $\langle V \rangle = e^*E/m' \times \tau = \mu^*E$
Note that two shaded area shall be the same, the mobility $\mu = e^* \tau / m'$.
\[ F = m_p \frac{d\epsilon}{dt} = e\epsilon \quad (4.10) \]

\[ v = \frac{e\epsilon_t}{m_p^*} \quad (4.11) \]

\[ v_{d|\text{peak}} = \left( \frac{e\tau_{cp}}{m_p^*} \right) \epsilon \quad (4.12a) \]

\[ \langle v_d \rangle = \frac{1}{2} \left( \frac{e\tau_{cp}}{m_p^*} \right) \epsilon \quad (4.12b) \]

\[ \mu_p = \frac{v_{dp}}{\epsilon} = \frac{e\tau_{cp}}{m_p^*} \quad (4.13) \]

Constant scattering (collision) time

Return to zero velocity

\[ \mu_n = \frac{e\tau_{cn}}{m_n^*} \quad (4.14) \]

statistic process

\[ \mu_L \propto T^{-3/2} \quad (4.15) \]

\[ \mu_1 \propto \frac{T^{+3/2}}{N_I} \quad (4.16) \]
Bulk mobility (T, NB)data

- Si : 1350 480
- GaAs: 8500 400
- Ge: 3900 1900
Figure 4.2 (a) Electron and (b) hole mobilities in silicon versus temperature for various doping concentrations. Inserts show temperature dependence for “almost” intrinsic silicon. *(From Pierret [8].)*
Figure 4.3 Electron and hole mobilities versus impurity concentrations for germanium, silicon, and gallium arsenide at $T = 300\text{K}$. (From Sze [14].)
\[
\frac{dt}{\tau} = \frac{dt}{\tau_t} + \frac{dt}{\tau_L} \quad (4.17)
\]

\[
\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L} \quad (4.18)
\]

\[
J_{drf} = e(\mu_n n + \mu_p p)\varepsilon = \sigma\varepsilon \quad (4.19)
\]

\[
\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)} \quad (4.20)
\]

p-type: \quad \sigma = e(\mu_n n + \mu_p p) \approx e\mu_pp \quad (4.23)

\[
R = \rho \frac{L}{A} = \frac{L}{(Wt)} = \left(\rho / t, \text{ sheet resistivity}\right)
\]

\[
\frac{L}{W}, \text{ square}
\]
Figure 4.4 Resistivity versus impurity concentration at $T = 300\text{K}$ in (a) silicon and (b) germanium, gallium arsenide, and gallium phosphide. (From Sze [14].)
Figure 4.6 Electron concentration and conductivity versus inverse temperature for silicon. (After Sze [14].)

Because $\mu$ decreases with increasing $T$.
Figure 4.7 Carrier drift velocity versus electric field for high-purity silicon, germanium, and gallium arsenide. (From Sze [14].)

\[ \frac{1}{2} m v_{th}^2 = \frac{3}{2} kT = \frac{3}{2} (0.0259) = 0.03885 \text{eV} \quad (4.26) \]
Figure 4.8 Energy – band structure for gallium arsenide showing the upper valley and lower valley in the conduction band. (From Sze [15].) NEGATIVE differential mobility
Figure 4.10 Electron concentration versus distance.

\[ n(x) \]

\[ n(0) \]

\[ n(-l) \]

\[ n(+l) \]

\[ x = -l \quad x = 0 \quad x = +l \]
Concentration after the scattering time

- $n(x)$
- $n(-l)$
- $n(0)$
- $n(l)$

Diffusion length

Electrons

Current

- $l$
- $0$
- $-l$
\[ F_n = \frac{1}{2} n(-l) v_{th} - \frac{1}{2} n(+l) v_{th} = \frac{1}{2} v_{th} \left[ n(-l) - n(+l) \right] \quad (4.27) \]

\[ F_n = \frac{1}{2} v_{th} \left\{ \left[ n(0) - l \frac{dn}{dx} \right] - \left[ n(0) + l \frac{dn}{dx} \right] \right\} \quad (4.28) \]

\[ F_n = -v_{th} l \frac{dn}{dx} \quad (4.29) \]

\[ J = -eF_n = ev_{th} l \frac{dn}{dx} \quad (4.30) \]

\[ J_{nx|\text{dif}} = eD_n \frac{dn}{dx} \quad (4.31) \]

\[ J_{px|\text{dif}} = -eD_p \frac{dp}{dx} \quad (4.32) \]
Total Current Density

- $J_n = en\mu_nE + eD_n \frac{dn}{dx}$
- $J_p = en\mu_pE - eD_p \frac{dp}{dx}$
Figure 4.12 Energy-band diagram for a semiconductor in thermal equilibrium with a nonuniform donor impurity concentration.
\[ E = -e\phi \quad (4.35a) \]
\[ \phi = \frac{-E}{e} \quad (4.35b) \]
\[ \phi_{Fn} = -\frac{E_{Fi} - E_F}{e} \quad (4.36) \]

\[ \text{E}_F \text{ is the reference energy} \]

\[ \varepsilon_x = -\frac{d\phi}{dx} \quad (4.37a) \]
\[ \varepsilon_x = -\frac{d\phi_{Fn}}{dx} = +\frac{1}{e} \frac{dE_{Fi}}{dx} \quad (4.37b) \]

\[ n_0 = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right) = N_d \quad (4.38) \]

**Induced electric field by doping**

\[ E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right) \quad (4.39) \]

**To see the sign by the physics**

\[ E_F - E_{Fi} = Cx \quad (4.40) \]

\[ N_d = N_d(x) = n_i \exp\left(\frac{Cx}{kT}\right) \quad (4.41) \]

\[ \varepsilon_x = +\frac{1}{e} \frac{dE_{Fi}}{dx} = -\left(\frac{kT}{e}\right) \frac{1}{N_d(x)} \frac{dN_d(x)}{dx} \quad (4.42) \]
\[ J_n = 0 = e n \mu_n \varepsilon_x + e D_n \frac{dn}{dx} \quad (4.43) \]

\[ J_n = 0 = e \mu_n N_d(x) \varepsilon_x + e D_n \frac{dN_d(x)}{dx} \quad (4.44) \]

\[ 0 = -e \mu_n N_d(x) \left( \frac{kT}{e} \right) \frac{1}{N_d(x)} \frac{dN_d(x)}{dx} + e D_n \frac{dN_d(x)}{dx} \quad (4.45) \]

\[ \frac{D_n}{\mu_n} = \frac{kT}{e} \quad (4.46a) \]

Einstein relation

\[ \frac{D_p}{\mu_p} = \frac{kT}{e} \quad (4.46b) \]

\[ \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e} \quad (4.47) \]
Figure 4.13 Electron-hole generation and recombination.

\[ G_{n0} = G_{p0} = R_{n0} = R_{p0} \] (4.50)
Table 4.3 Relevant notation used in Section 4.4

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>$n_0, p_0$</td>
<td>Thermal-equilibrium electron and hole concentrations (independent of time and also usually position).</td>
</tr>
<tr>
<td>$n, p$</td>
<td>Total electron and hole concentrations (may be functions of time and / or position).</td>
</tr>
<tr>
<td>$\delta n = n - n_0$</td>
<td>Excess electron and hole concentrations (may be functions of time and / or position).</td>
</tr>
<tr>
<td>$\delta p = p - p_0$</td>
<td>Excess electron and hole concentrations (may be functions of time and / or position).</td>
</tr>
<tr>
<td>$g'_n, g'_p$</td>
<td>Excess electron and hole generation rates.($\delta n$)</td>
</tr>
<tr>
<td>$R'_n, R'_p$</td>
<td>Excess electron and hole recombination rates.</td>
</tr>
<tr>
<td>$\tau_{n0}, \tau_{p0}$</td>
<td>Excess minority-carrier electron and hole lifetimes.</td>
</tr>
</tbody>
</table>
Figure 4.15 Recombination of excess carriers reestablishing thermal equilibrium.

Excess carrier can be generated by the photo (reverse process)
\[ g'_n = g'_p \quad (4.51) \text{ band to band generation} \]

\[ p = p_0 + \delta p \quad (4.52b) \]

\[ n = n_0 + \delta n \quad (4.52a) \]

\[ R'_n = R'_p \quad (4.53) \text{ band to band recom.} \]

\[ \frac{dn(t)}{dt} = \alpha_r \left[ n_i^2 - n(t)p(t) \right] \quad (4.54) \]

\[ \alpha_r n_i^2 : \text{Generation rate at thermal equil.} \]

\[ n(t) = n_0 + \delta n(t) \quad (4.55a) \]

\[ p(t) = p_0 + \delta p(t) \quad (4.55b) \]
\[
\frac{dn(t)}{dt} = \alpha_r \left[ n_i^2 - (n_0 + \delta n(t))(p_0 + \delta p(t)) \right] \\
= -\alpha_r \delta n(t) \left[ (n_0 + p_0) + \delta n(t) \right] \tag{4.56}
\]

\[
\frac{d(\delta n(t))}{dt} = -\alpha_r p_0 \delta n(t) \tag{4.57}
\]

for p type and low level injection \( \delta n < p_0 \)

\[
\delta n(t) = \delta n(0) e^{-\alpha_r p_0 t} = \delta n(0) e^{-t/\tau_0} \tag{4.58}
\]

\[
\tau_0 = (\alpha_r p_0)^{-1} : \text{Excess minority carrier lifetime}
\]

Direct band to band recombination

\[
R'_n = \frac{-d(\delta n(t))}{dt} = +\alpha_r p_0 \delta n(t) = \frac{\delta n(t)}{\tau_0} \tag{4.59}
\]

\[
R'_n = R'_p = \frac{\delta n(t)}{\tau_n} \tag{4.60}
\]

R is positive

\[
R'_n = R'_p = \frac{\delta n(t)}{\tau_{p0}} \tag{4.61}
\]
Figure 4.16 **Recombination** via a trapping center: an electron is trapped and then a hole is trapped; or a hole is trapped and then an electron is trapped; or an electron is trapped and then the electron falls into an empty state (hole). **ALL are equivalent**

Figure 4.17 **Generation** via a trapping center: an electron is elevated from the valence band into the trap creating a free hole and then the electron is elevated into the conduction band creating a free electron.
Figure 4.18 (a) Auger recombination involving two holes and (b) Auger recombination involving two electrons. $R= np^2$

For n-type, which is preferred?

Important in direct bandgap materials with high doping concentrations
Figure 4.19 (a) Recombination in a direct bandgap semiconductor (such as GaAs) with no change in momentum required. (b) Recombination in an indirect bandgap semiconductor (such as Si) with a change in momentum required.

Momentum consideration:

Photon has very low momentum w.r.t. energy band due to large wavelength (~ 1000 nm)

Director band gap: emit photon
Indirect bandgap: emit phonon.
**Hall effect**

Distinguish semiconductor types

Majority-carrier concentration

Majority-carrier mobility

Figure 4.20 (a) Geometry for measuring the Hall effect.
\[
F = qv \times B \quad (4.62) \quad F = q[\varepsilon + v \times B] = 0 \quad (4.63a) \quad q \varepsilon_y = qv_x B_z \quad (4.63b)
\]
\[
V_H = +\varepsilon_H W \quad (4.64) \quad V_H = v_x W B_z \quad (4.65) \quad v_{dx} = \frac{J_x}{ep} = \frac{I_x}{(ep)(Wd)} \quad (4.66)
\]
\[
V_H = \frac{I_x B_z}{epd} \quad (4.67) \quad p = \frac{I_x B_z}{ed V_H} \quad (4.68) \quad V_H = \frac{I_x B_z}{ned} \quad (4.69)
\]

Hall voltage
\[
n = -\frac{I_x B_z}{ed V_H} \quad (4.70)
\]
\[
J_x = ep \mu_p \varepsilon_x \quad (4.71)
\]
\[
\frac{I_x}{Wd} = \frac{ep \mu_p V_x}{L} \quad (4.72)
\]

Measure the hole Hall mobility
\[
\mu_p = \frac{I_x L}{ep V_x Wd} \quad (4.73)
\]
\[
\mu_n = \frac{I_x L}{en V_x Wd} \quad (4.74)
\]