Figure 5.1 (a) Simplified geometry of a pn junction; (b) Doping profile of an ideal uniformly doped pn junction.

Figure 5.2 The space charge region, the electric field, and the forces acting on the charged carriers.
Figure 5.3 Energy-band diagram of a pn junction in thermal equilibrium (constant Fermi level) showing the energy bands in the neutral p and n regions. Three definitions of the built-in potential Barrier are shown – all being equivalent.
\[ V_{bi} = |\phi_{Fn}| + |\phi_{Fp}| \]  (5.1)

\[ n_0 = N_c \exp \left[ \frac{-(E_c - E_F)}{kT} \right] \]  (5.2)

\[ n_0 = n_i \exp \left( \frac{E_F - E_{Fi}}{kT} \right) \]  (5.3)

\[ e\phi_{Fn} = E_F - E_{Fi} \]  (5.4)

\[ n_0 = n_i \exp \left[ \frac{+(e\phi_{Fn})}{kT} \right] \]  (5.5)

\[ \phi_{Fn} = \frac{kT}{e} \ln \left( \frac{N_d}{n_i} \right) \]  (5.6)

\[ p_0 = N_a = n_i \exp \left( \frac{E_{Fi} - E_F}{kT} \right) \]  (5.7)

\[ e\phi_{Fp} = E_F - E_{Fi} \]  (5.8)

\[ \phi_{Fp} = \frac{-kT}{e} \ln \left( \frac{N_a}{n_i} \right) \]  (5.9)

\[ V_{bi} = \frac{kT}{e} \ln \left( \frac{N_a N_d}{n_i^2} \right) = V_t \ln \left( \frac{N_a N_d}{n_i^2} \right) \]  (5.10)

\[ V_{bi} = 60meV \left[ \log_{10} \left( \frac{N_D^+}{n_i} \right) + \log_{10} \left( \frac{N_A^-}{n_i} \right) \right] \]
\[
\frac{d^2 \phi(x)}{dx^2} = -\frac{\rho}{\varepsilon_s} = \frac{d\varepsilon(x)}{dx} \quad (5.11)
\]

\[\rho(x) = -eN_a \quad -x_p < x < 0 \quad (5.12a) \quad \rho(x) = eN_d \quad 0 < x < x_n \quad (5.12b)\]

\[\varepsilon = \int \frac{\rho(x)}{\varepsilon_s} \, dx = -\int \frac{eN_a}{\varepsilon_s} \, dx = \frac{-eN_a}{\varepsilon_s} x + C_1 \quad (5.13)\]

\[\varepsilon = \frac{-eN_a}{\varepsilon_s} (x + x_p) \quad -x_p \leq x \leq 0 \quad (5.14) \text{ E-field is zero at } -x_p\]

\[\varepsilon = \int \frac{eN_d}{\varepsilon_s} \, dx = \frac{eN_d}{\varepsilon_s} x + C_2 \quad (5.15)\]

\[\varepsilon = \frac{-eN_d}{\varepsilon_s} (x_n - x) \quad 0 \leq x \leq x_n \quad (5.16)\]

\[N_a x_p = N_d x_n \quad (5.17)\]
Figure 5.4 The space charge density in a uniformly doped pn junction assuming the abrupt junction approximation.
Figure 5.5 Electric field in the space charge region of a uniformly doped pn junction. The linear $\varepsilon$-field versus distance is a result of a uniformly doped junction.
\[ \phi(x) = -\int \varepsilon(x) dx = \int \frac{eN_a}{\varepsilon_s} (x + x_p) dx \quad (5.18) \]

\[ \phi(x) = \frac{eN_a}{\varepsilon_s} \left( \frac{x^2}{2} + x_p x \right) + C_1 \quad (5.19) \]

\[ C_1' = \frac{eN_a}{2\varepsilon_s} x_p^2 \quad (5.20) \quad \text{set } \phi = 0 \text{ at } x = -x_p \]

\[ \phi(x) = \frac{eN_a}{2\varepsilon_s} (x + x_p)^2 \quad (-x_p \leq x \leq 0) \quad (5.21) \quad \text{potential is zero at -xp} \]

\[ \phi(x) = \int \frac{eN_d}{\varepsilon_s} (x_n - x) dx \quad (5.22) \]

\[ \phi(x) = \frac{eN_d}{\varepsilon_s} \left( x_n x - \frac{x^2}{2} \right) + C_2' \quad (5.23) \]

\[ C_2' = \frac{eN_a}{2\varepsilon_s} x_p^2 \quad (5.24) \quad \phi \text{ Continuous at } x=0 \]

\[ \phi(x) = \frac{eN_d}{\varepsilon_s} \left( x_n x - \frac{x^2}{2} \right)^2 + \frac{eN_a}{2\varepsilon_s} x_p^2 \quad (0 \leq x \leq x_n) \quad (5.25) \]

\[ V_{bi} = |\phi(x = x_n)| = \frac{e}{2\varepsilon_s} \left( N_d x_n^2 + N_a x_p^2 \right) \quad (5.26) \quad \text{superposition}^{128} \]
Figure 5.7 Energy-band diagram of a pn junction in thermal equilibrium.
\[ x_p = \frac{N_d x_n}{N_a} \]  \hspace{1cm} (5.27)

\[ x_n = \left[ \frac{2\varepsilon_S V_{bi}}{e} \left( \frac{N_a}{N_d} \right) \left( \frac{1}{N_a + N_d} \right) \right]^{1/2} \]  \hspace{1cm} (5.28)

\[ x_p = \left[ \frac{2\varepsilon_S V_{bi}}{e} \left( \frac{N_d}{N_a} \right) \left( \frac{1}{N_a + N_d} \right) \right]^{1/2} \]  \hspace{1cm} (5.29)

\[ W = x_n + x_p \]  \hspace{1cm} (5.30)

\[ W = \left[ \frac{2\varepsilon_S V_{bi}}{e} \left( \frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2} \]  \hspace{1cm} (5.31)

Charge conservation and superposition can solve this
Figure 5.8 Energy-band diagram of a pn junction under reverse bias (n region positive with respect to p region).
Figure 5.9 A pn junction with an applied reverse-bias voltage, showing the direction of the electric field induced by $V_R$ and the direction of the zero-biased space charge electric field.

\[
W = \left[ \frac{2 \varepsilon_S (V_{bi} + V_R)}{e} \left( \frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2}
\]  \hspace{1cm} (5.34a)

\[
x_p = \left[ \frac{2 \varepsilon_S (V_{bi} + V_R)}{e} \left( \frac{N_d}{N_a} \right) \left( \frac{1}{N_a + N_d} \right) \right]^{1/2}
\]  \hspace{1cm} (5.34b)

\[
x_n = \left[ \frac{2 \varepsilon_S (V_{bi} + V_R)}{e} \left( \frac{N_a}{N_d} \right) \left( \frac{1}{N_a + N_d} \right) \right]^{1/2}
\]  \hspace{1cm} (5.34c)
Figure 5.10 Differential change in the space charge width with a differential change in reverse-bias voltage for a uniformly doped pn junction. Also shown are the additional charges uncovered by an increase in reverse-bias voltage.
\[ C' = \frac{dQ'}{dV_R} \]  
(5.38)

\[ dQ' = eN_a dx_n = eN_a dx_p \]  
(5.39)

\[ x_n = \left\{ \frac{2\varepsilon_S(V_{bi} + V_R)}{e} \left( \frac{N_a}{N_d} \right) \left( \frac{1}{N_a + N_d} \right) \right\}^{1/2} \]  
(5.40)

\[ C' = \frac{dQ'}{dV_R} = eN_d \frac{dx_n}{dV_R} \]  
(5.41)

\[ C' = \left[ \frac{e\varepsilon_S N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right]^{1/2} \]  
(5.42)

\[ C' = \frac{\varepsilon_S}{W} \]  
(5.43)
For $\text{Na} \gg \text{Nd}$

$$W \approx \left[ \frac{2\varepsilon_s(V_{bi} + V_R)}{eN_d} \right]$$  (5.44)

$$x_p \ll x_n$$  (5.45)

$$W \approx x_n$$  (5.46)

$$C' \approx \left[ \frac{2\varepsilon_s N_d}{2(V_{bi} + V_R)} \right]^{1/2}$$  (5.47)

$$\left( \frac{1}{C'} \right)^2 = \frac{2(V_{bi} + V_R)}{e\varepsilon_s N_d}$$  (5.48)

Figure 5.12 $(1/C')^2$ versus $V_R$ of a uniformly doped pn junction.
Figure 5.13 (a) Energy-band diagram of a metal and semiconductor before contact; (b) ideal energy-band diagram of a metal-n-semiconductor junction for $\phi_m > \phi_s$.

\[ \phi_{B0} = (\phi_m - x) \]  
(5.49)

\[ V_{bi} = \phi_{B0} - \phi_n \]  
(5.50)

\[ x_n = W \]
Figure 5.14 Ideal energy-band diagram of a metal-semiconductor rectifying junction under reverse bias.

\[ x_n = \left[ \frac{2\varepsilon_s (V_{bi} + V_R)}{eN_d} \right]^{1/2} \quad (5.51) \]

\[ C' = \left[ \frac{e\varepsilon_s N_d}{2(V_{bi} + V_R)} \right]^{1/2} \quad (5.52) \]
Figure 5.15 The pn junction with an applied forward-bias voltage showing the direction of the electric field induced by $V_D$ and the direction of the electric field of the zero-biased pn junction.

Figure 5.16 Energy-band diagram of the forward biased pn junction showing the reduction of the barrier.
Figure 5.17 Steady-state minority-carrier concentrations in a pn junction under forward bias.

\[ I_D = I_S \left[ \exp \left( \frac{V_D}{V_t} \right) - 1 \right] \] (5.53)

\[ I_D \approx I_S \exp \left( \frac{V_D}{V_t} \right) \] (5.54)

Figure 5.18 Plot of ideal I-V characteristics of a pn junction diode.
Figure 5.19 Energy-band diagram of a forward-biased metal-semiconductor rectifying contact showing current directions. Also shown is the circuit symbol of a Schottky Barrier diode.
\[ J = J_{s \rightarrow m} - J_{m \rightarrow s} \]  
(5.55)

\[ J = \left[ A^* T^2 \exp \left( -\frac{e\phi_{B0}}{kT} \right) \right] \exp \left( \frac{eV_D}{kT} \right) - 1 \]  
(5.56)

\[ A^* = \frac{4\pi e m_n^* k^2}{\hbar^3} \]  
(5.57)

\[ J = J_{ST} \left[ \exp \left( \frac{eV_D}{kT} \right) - 1 \right] \]  
(5.58)
Figure 5.20 Comparison of forward-bias $I-V$ characteristics between a Schottky diode and a pn diode. Rectifier
Figure 5.21 Ideal energy-band diagram (a) before contact and (b) after contact for a metal-n-semiconductor junction for $\phi_m < \phi_s$. 
Figure 5.22 Ideal energy-band diagram of a metal-n-semiconductor ohmic contact (a) with a positive voltage applied to the metal and (b) with a positive voltage applied to the semiconductor.
Figure 5.23 Ideal energy-band diagram (a) before contact and (b) after contact for a metal-p-semiconductor junction for $\phi_m > \phi_s$. 
Figure 5.24 Energy-band diagram of a heavily doped n-semiconductor-to-metal junction.

Depletion width decreases, probability of tunneling increases
Figure 5.25 Impurity concentrations of a pn junction with a nonuniformly doped p region.

Figure 5.26 Space charge density density in a linearly graded pn junction.
\[
\rho(x) = eax
\]

\[
\varepsilon = \int_{\varepsilon_s}^{eax} \frac{dx}{\varepsilon_s} = \frac{ea}{2\varepsilon_s} \left( x^2 - x_0^2 \right) \tag{5.61}
\]

\[
\phi(x) = -\int \varepsilon dx \tag{5.62}
\]

\[
\phi(x) = \frac{-ea}{2\varepsilon_s} \left( \frac{x^3}{3} - x_0^2 x \right) + \frac{ea}{3\varepsilon_s} x_0^3 \tag{5.63}
\]

set \( \phi = 0 \) at \( x = -x_0 \)

\[
\phi(x_0) = \frac{2}{3} \frac{e \alpha x_0^3}{\varepsilon_s} = V_{bi} \tag{5.64}
\]

\[
V_{bi} = V_t \ln \left[ \frac{N_d(x_0)N_a(-x_0)}{n_i^2} \right] \tag{5.65}
\]
\[ N_d(x_0) = ax_0 \quad (5.66a) \]
\[ N_a(-x_0) = ax_0 \quad (5.66b) \]

\[ V_{bi} = V_i \ln \left( \frac{ax_0}{n_i} \right)^2 \quad (5.67) \]

\[ x_0 = \left[ \frac{3 \varepsilon_s}{2ea} \left( V_{bi}(x_0) + V_R \right) \right]^{1/3} \quad (5.68) \]

\[ C' = \frac{dQ'}{dV_R} = (eax_0) \frac{dx_0}{dV_R} \quad (5.69) \]

\[ C' = \left[ \frac{ea \varepsilon_s^2}{12(V_{bi} + V_R)} \right]^{1/3} \quad (5.70) \]
Figure 5.27 Differential change in space charge width with a differential change in reverse-bias voltage for a linearly graded pn junction.
Figure 5.28 Generalized doping profiles of a one-sided p\textsuperscript{+}n junction. 
(From Sze [15].)
\[ N = Bx^m \quad (5.71) \]

\[ C' = \left[ \frac{eB\varepsilon_s^{(m+1)}}{(m+2)(V_{bi} + V_R)} \right]^{1/(m+2)} \quad (5.72) \]

\[ f_r = \frac{1}{2\pi \sqrt{LC}} \quad (5.73) \]

\[ C = C_0 (V_{bi} + V_R)^{-1/(m+2)} \quad (5.74) \]

\[ C \propto V^{-2} \quad (5.75) \]

\[ \frac{1}{m+2} = 2 \quad (5.76a) \]

\[ m = -\frac{3}{2} \quad (5.76b) \]